

# Solución

① a)  $\sum_{k=1}^{\infty} k(k-1)a^k$

a) es una serie infinita, converge.

b) Estrategia: Manipulación matemática, aplicar fórmula.

$\sum_{k=1}^{\infty} k(k-1)a^k$  \* Voy a multiplicar todo, después separo por la resta.

$$\sum_{k=1}^{\infty} k^2 a^k - k a^k = \sum_{k=1}^{\infty} k^2 a^k - \sum_{k=1}^{\infty} k a^k$$

Elimino el término  $k=0$ , el cual es 0 para este caso.

$$\left[ \sum_{k=0}^{\infty} k^2 a^k - 0^2 a^0 \right] - \left[ \sum_{k=0}^{\infty} k a^k - 0 a^0 \right] =$$

$$\frac{a(a+1)}{(1-a)^3} - 0 - \frac{a}{(1-a)^2} + 0 = \frac{a(a+1)}{(1-a)^3} - \frac{a}{(1-a)^2}$$

$$\sum_{k=1}^{\infty} k(k-1)a^k = \frac{a(a+1) - a(1-a)}{(1-a)^3} = \frac{a^2 + a - a + a^2}{(1-a)^3} = \underline{\underline{\frac{2a^2}{(1-a)^3}}}$$

b)  $a = \frac{1}{3}$ ,  $\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k$

a) serie geométrica, converge.

b) esa  $a$ , debe ser  $a$  para sustituir en la fórmula.

c) Estrategia: usar fórmula.



$$\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k = \frac{1}{1-a} = \frac{1}{1-1/3} = \underline{\underline{\frac{3}{2}}}$$

a)  $a = \frac{1}{2}, \sum_{k=0}^{\infty} \frac{k}{2^k}$

a) serie converge.

b) a debe ser la a para sustituir en la fórmula

c) Estrategia: Manipular la serie, aplicar fórmula

$$\begin{aligned} \sum_{k=0}^{\infty} \frac{k}{2^k} &= \sum_{k=0}^{\infty} k \left(\frac{1}{2^k}\right) = \sum_{k=0}^{\infty} k \left(\frac{1}{2}\right)^k = \frac{a}{(1-a)^2} \\ &= \frac{1/2}{(1-1/2)^2} = \underline{\underline{2}} \end{aligned}$$

d)  $\sum_{p=m}^{\infty} p \alpha^p (\alpha^2 + 1)$

a) la serie converge si  $|\alpha| < 1$ ,

b) la serie empieza en m, T-T

c) Estrategia: Cambio de variable, manipulación, aplicar fórmula

$$\sum_{p=m}^{\infty} p \alpha^p (\alpha^2 + 1) = \sum_{k=0}^{\infty} (k+m) \alpha^{k+m} (\alpha^2 + 1)$$

$$\begin{aligned} k=0 \\ p-m=0 \\ p-m=k \\ p=k+m \end{aligned}$$





$$(\alpha^2 + 1) \sum_{k=0}^{\infty} (k+m) \alpha^k \alpha^m = (\alpha^2 + 1) \alpha^m \sum_{k=0}^{\infty} k \alpha^k + m \alpha^m \sum_{k=0}^{\infty} \alpha^k =$$

$$(\alpha^2 + 1) \alpha^m \left( \sum_{k=0}^{\infty} k \alpha^k + m \sum_{k=0}^{\infty} \alpha^k \right) = (\alpha^2 + 1) \alpha^m \left( \frac{\alpha}{(1-\alpha)^2} + m \left( \frac{1}{1-\alpha} \right) \right)$$

$$\sum_{m=0}^{\infty} p \alpha^m (\alpha^2 + 1) = \frac{\alpha^m (\alpha^2 + 1) (m(1-\alpha) + \alpha)}{(1-\alpha)^2}$$

2) Usando una serie geométrica prueba que  $0.\overline{33} = 1/3$

a) T-T, Estrategia: Convertir  $0.\overline{33}$  en serie, aplicar fórmula.

$$0.3\overline{33} = \frac{1}{3}$$

$$0.3 + 0.03 + 0.003 + \dots = \frac{1}{3}$$

$$\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots = \frac{1}{3}$$

$$3 \left( \frac{1}{10} \right) + 3 \left( \frac{1}{10^2} \right) + 3 \left( \frac{1}{10^3} \right) + \dots = \frac{1}{3}$$

$$3 \left( \frac{1}{10} \right)^1 + 3 \left( \frac{1}{10} \right)^2 + 3 \left( \frac{1}{10} \right)^3 + \dots = \frac{1}{3}$$

$$\sum_{k=1}^{\infty} 3 \left( \frac{1}{10} \right)^k = 3 \left( \frac{1}{10} \right)^0 = 3 \left( \frac{1}{1-\frac{1}{10}} \right) - 3 = \frac{1}{3}$$

$$\left( \frac{3}{1} \cdot \frac{10}{9} \right) - \frac{27}{9} = \frac{1}{3}$$

$$\frac{3}{9} = \frac{1}{3} \quad \text{Q.E.D.} \quad \text{||}$$



3) Usando una serie geométrica demuestra  $0.\overline{99} = 1$

a) T-T, se parece al 2. Estrategia, la misma.

$$0.\overline{999} = 1$$

$$0.9 + 0.09 + 0.009 + \dots = 1$$

$$\frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \dots = 1$$

$$9\left(\frac{1}{10}\right) + 9\left(\frac{1}{100}\right) + 9\left(\frac{1}{1000}\right) + \dots = 1$$

$$9\left(\frac{1}{10}\right) + 9\left(\frac{1}{10^2}\right) + 9\left(\frac{1}{10^3}\right) + \dots = 1$$

$$\sum_{k=1}^{\infty} 9\left(\frac{1}{10}\right)^k = 1$$

$$\sum_{k=0}^{\infty} 9\left(\frac{1}{10}\right)^k - 9\left(\frac{1}{10}\right)^0 = 1$$

$$9\left(\frac{1}{1 - 1/10}\right) - 9 = 1$$

$$\left(\frac{9}{1} \cdot \frac{10}{9}\right) - \frac{81}{9} = 1$$

$$\frac{9}{9} = 1 \quad \text{QED} \quad \nabla$$

4) Obtén las formas cerradas de:

$$\sum_{k=1}^n \frac{1}{k(k+1)} \quad \text{y} \quad \sum_{k=1}^{\infty} \frac{1}{k(k+1)}$$

a) T-T, No es de las del formulario.

b) Estrategia: tomar el tip, expandir para la de N





Sacar el límite a la  $n$  para obtener la de  $\infty$

$$\begin{aligned}\sum_{k=1}^n \frac{1}{k(k+1)} &= \sum_{k=1}^n \frac{1}{k} - \frac{1}{k+1} = \left(\frac{1}{1} - \frac{1}{1+1}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots \\ &= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots + \left(\frac{1}{n-1} - \frac{1}{n}\right) + \left(\frac{1}{n} - \frac{1}{n+1}\right) \\ &= \frac{1}{1} - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \underline{\underline{\frac{n}{n+1}}}\end{aligned}$$

$$\begin{aligned}\sum_{k=1}^{\infty} \frac{1}{k(k+1)} &= \lim_{n \rightarrow \infty} \left\{ \frac{n}{n+1} \right\} = \frac{\infty}{\infty} \quad \parallel \\ &\quad \lim_{n \rightarrow \infty} \left\{ \frac{1}{1+\frac{1}{n}} \right\} = \underline{\underline{1}} \quad \text{L'Hopital}\end{aligned}$$

